# CASIO EDUCATION

CASIO Education Workbook Series

ACTIVITY SAMPLES

with the

CASIO *fx-9750GII*

WHAT'S INSIDE:

- Algebra I: Fun with Functions
- Algebra II: Saving for a Rainy Day
- Geometry: Quite the Quadrilateral
- Statistcs: The Cost of Car Insurance
- Pre-Calculus: Area of a Triangle and Circular Sector
- Calculus: A Graphical Look at Continuity

Fun With Functions   Teacher Notes
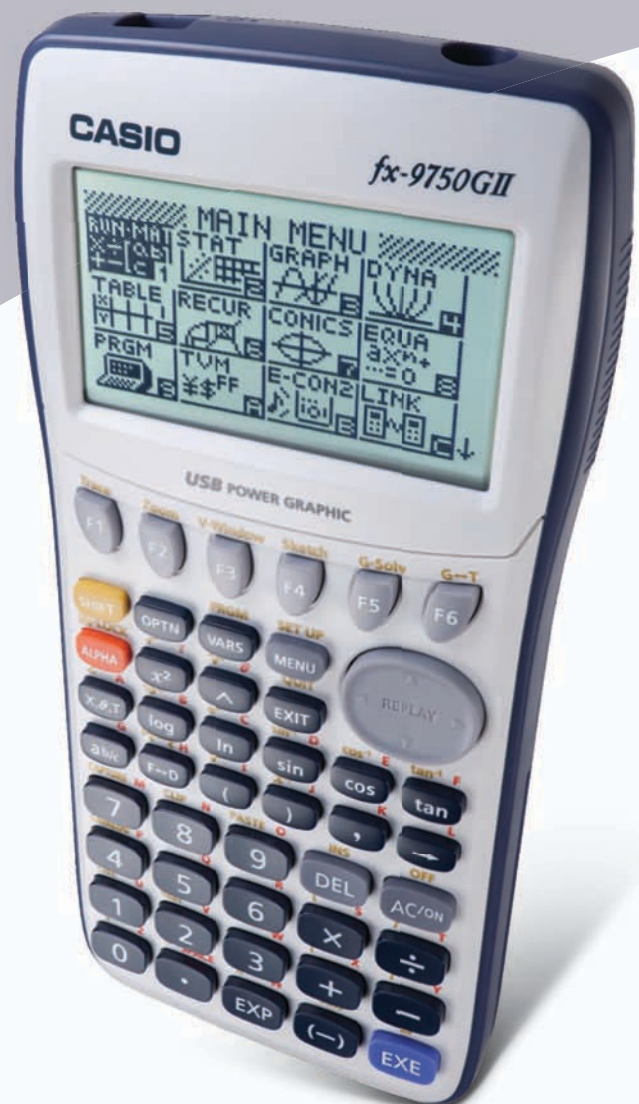
Topic: Functions and Function Notation

NCTM Standard:
- Represent, analyze, and generalize a variety of patterns with tables, graphs, words, and, when possible, symbolic rules.
- Relate and compare different forms of representation for a relationship.
- Identify functions as linear or nonlinear and contrast their properties from tables, graphs, or equations.

Objective
> The student will be able to use the Casio *fx-9750GII* to input data relating to functions as well as to evaluate functions using function notation.

Getting Started
> Being able to understand functions in various forms empowers students to see patterns in relationships and make connections with functions to their everyday lives. Begin this activity by looking at various tables and gain an understanding of how that data relates to its graph on a coordinate plane. Students should be able to examine a table and/or a graph and be able to determine any specific trends or characteristics about that data set.

Prior to using this activity:
- Students should be able to graph points on a coordinate plane.
- Students should be able to read and interpret data presented in a table and graph.
- Students should be familiar with all key strokes involved in entering data into a table and entering a function into the graph editor window.
- Students should understand the formula for a linear function.
- Students should be able to correctly identify the independent (x) and dependent (y) variables.

Ways students can provide evidence of learning:
- When given a function, students can state and explain whether that function is increasing or decreasing.
- When given a function, students can evaluate it for a specific value of *x* or *y*.

Common calculator or content errors students might make:
- Students may incorrectly set the viewing window when showcasing a function.
- Students may enter data into a table incorrectly by switching the x- and y-coordinates.

Fun With Functions   "How To"

The following will demonstrate how to store a value for a variable, input a function to generate a table, and enter data into a table to construct a graph with the Casio *fx-9750 GII*.
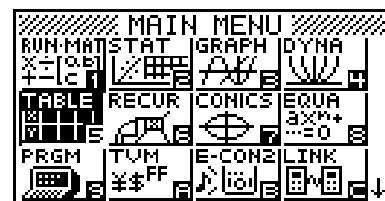
To store a value for a variable:

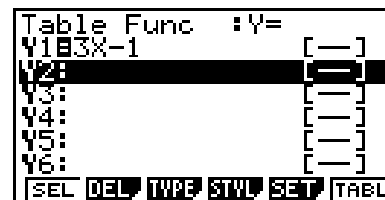1.   To store 5 for the variable *x*, input:

      [5] [→] [X,θ,T] [EXE] .

To input a function and generate a table:

1.   From the Main Menu, highlight the TABLE icon and press [EXE] or [5].

2.   To enter a function such as 3x – 1,

      input: [3] [X,θ,T] [–] [1] [EXE] .

3.   The equal sign to the right of **Y1:** is highlighted to indicate this function is active.

4.   To generate a table of values for the selected function, press [F6] (TABL).
      The default x-values for the table menu is x starts 1, ends at 5 and increases by steps of 1.

5.   To navigate through the table, use the replay pad
      ◀ ▶ ▲ ▼ .

To adjust the values in a table:

1.   Use [EXIT] to return to the TABLE home screen. Press [F5] (SET) to change the default table values.
      Enter 0 for the start value, 20 for the end value and a step value of 1, pressing [EXE] after each entry.
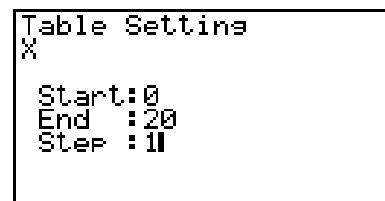
2. Press [EXIT] then [F6] (TABL) to display the table.

3. To display a corresponding y-value for a specific x-value, highlight any x-value and enter the desired value.

   To display the corresponding y-value when x = 18, input [1] [8] [EXE].

   Note: You do not need to change the tables settings; you could just enter all given x-values and create a custom table.

To enter data into a list and graph the data:

1. From the Main Menu, highlight the STAT icon and press [EXE] or [2].

2. In List 1, input: [1] [EXE] [2] [EXE] [3] [EXE].

3. Press [▶] to move the cursor to List 2.

4. In List 2, input: [3] [EXE] [5] [EXE] [7] [EXE].

5. Press [F1] (GRPH), then [F1] to select Graph 1 (the default graph type is a scatterplot).

6. Press [F1] (CALC) for regression options.

7. Since the data appears linear, press [F2] (X) to calculate linear regression (line of best fit).

8. Press [F1] (ax +b) for slope-intercept form of a line. Substitute the a- and b-values displayed into the given formula.

   When the correlation coefficient, r, equals 1, you have a perfect regression.

Fun With Functions   Activity
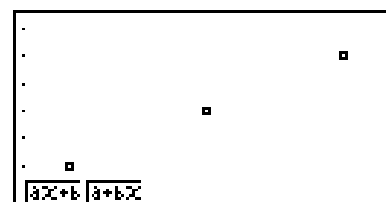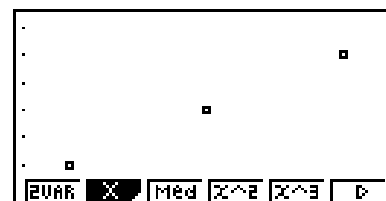
Functions help establish various types of numeric patterns, based upon whether those functions are linear, quadratic, cubic, etc. Building a strong foundation in Algebra includes a comprehensive study of linear functions. Functions are a rule used to calculate values. Functions are written using a specific notation called function notation. Each function has an independent and a dependent variable. The independent variable is the value you get to choose or control. The dependent variable is the value created when the independent variable is plugged into the function. Another name for the independent variable is the "input" and for the dependent variable is the "output". We will define a series of coordinate points as a relation.

In this activity, we will explore how to assign a single value to a variable and evaluate a given function. We will also explore how to input a function and generate a table of values as well as enter points in a data set and determine the function.

Functions can be expressed in these different forms:

1.   The **Slope-Intercept Form of a Line** is defined as **y = mx + b**; where *m*

       represents the slope of the line, *b* represents the y-intercept, *x* represents the

       independent variable and *y* represents the dependent variable.

2.   The **Standard Form of a Line** is defined as **Ax + By = C** where **A**, **B**, and **C**

       are integers and **x** represents the independent variable while **y** represents the

       dependent variable.

3.   f(x) is often described as function notation. In this example, where

       f(x) = 5x – 3, *x* represents the independent variable and f(x) is synonymous

       with *y*, representing the dependent variable.

Remember when graphing a function, it must pass the Vertical Line Test. A function is defined as a relation where for every one *x*-value, there is one and only one *y*-value. When looking at the graph, if any two points appear directly above each other, the graph fails the vertical line test and thus, the relation is not a function.

Questions

1. Given the function $f(x) = 3x + 7$, evaluate the function at $f(4)$.

    _____

2. Given the function $f(x) = -2x - 5$, evaluate the function where $-5 \leq x \leq -1$.
    (Let x = set of integers)

    _____

3. Given the function $f(x) = 2x^2 + 6x - 5$, evaluate the function at $f(-2)$.

    _____

4. Enter the data into the calculator and determine the linear function.

List 1	List 2
1	2
2	5
3	8
4	11

    _____

5. Enter the data into the calculator and determine the linear function.

List 1	List 2
1	−1
2	17
5	32
8	47

    _____

6.   Enter the data into the calculator and determine the linear function.

List 1	List 2
0	8
2	6
5	3
2	0

_____

7.   Lauren works as a babysitter to earn some extra money. She charges her
        customers seven dollars an hour. Write a function to determine the amount
        of money Lauren will earn if she works x hours. How much money will she
        earn if she works 4 hours? 7 hours? 11 hours?

_____

_____

8.   A cell phone company charges its customers $30 per month for phone calls
        plus an additional charge of seven cents per text message after the first 50
        text messages. Write a function that accurately models how much money
        you will spend per month with this plan. How much money will you spend if
        you send 50 text messages per month? 100 text messages per month? 225
        text messages per month? 500 text messages per month?

_____

_____

_____

Extension

1.   Create a data set that models a function which is non-linear.

_____

_____

2.   Write a non-linear function and evaluate that function where $-5 \leq x \leq 5$.

_____

_____

Solutions

1.   f(4) = 19

```
4→X
                4
3X+7
               19


▶MAT
```

2.   f(-5) = 5, f(-4) = 3, f(-3) = 1, f(-2) = -1, f(-1) = -3

```
Table Func   :Y=
Y1■-2X-5        [—]
Y2:            [—]
Y3:            [—]
Y4:            [—]
Y5:            [—]
Y6:            [—]
SEL DEL TYPE STYL SET TABL
```

```
Table Setting
X

 Start:-5
 End  :-1
 Step :1
```

```
    X    Y1
┌──────────
│   -5    5
│   -4    3
│   -3    1
│   -2   -1
│              -5
FORM DEL ROW EDIT G·CON G·PLT
```

```
    X    Y1
┌──────────
│   -4    3
│   -3    1
│   -2   -1
│   -1   -3
│              -1
FORM DEL ROW EDIT G·CON G·PLT
```

3.   f(-2)= -9

```
-2→X
                -2
2X²+6X-5
                -9


▶MAT
```

4.   The function is: y = 3x – 1 or f(x) = 3x – 1

```
   List 1 List 2 List 3 List 4
SUB
 1     1     2
 2     2     5
 3     3     8
 4     4    11
                          1
GRPH CALC TEST INTR DIST  ▷
```

```



           □
        □
     □
  □
CALC DefG
```
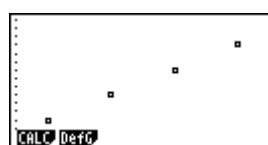
```
LinearReg(ax+b)
  a =3
  b =-1
  r =1
  r²=1
  MSe=0
y=ax+b
             COPY DRAW
```

5.   The function is y = 5x + 7 or f(x) = 5x + 7

```
   List 1 List 2 List 3 List 4
SUB
 1    -1     2
 2     2    17
 3     5    32
 4     8    47
                         -1
GRPH CALC TEST INTR DIST  ▷
```

```
                    □
                □
            □
 □
CALC DefG
```
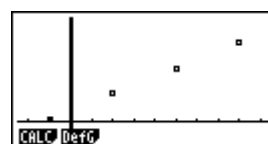
```
LinearReg(ax+b)
  a =5
  b =7
  r =1
  r²=1
  MSe=0
y=ax+b
             COPY DRAW
```
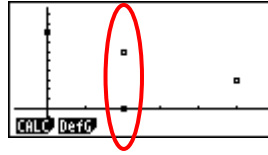
6.   This relation fails the vertical line test as evident by the two coordinates directly above each other. Since the relation fails the vertical line test, the relation is not a function.

7.   The function is y = 7x or f(x) = 7x. If Lauren works four hours, she will earn $28. If Lauren works seven hours, she will earn $49. If Lauren works 11 hours, she will earn $77.

8.   The function is y = .07(x - 50) + 30 or f(x) = .07(x - 50). For 50 text messages per month, you will spend $30. For 100 text messages per month, you will spend $33.50. For 225 text messages per month, you will spend $42.25 and for 500 text messages per month, you will spend $61.50.

Extensions

1.   Answers may vary.

2.   Answers may vary.

Saving for a Rainy Day   Teacher Notes

Topic Area: Patterns and Functions – Algebraic Thinking

NCTM Standard:
- Understand patterns, relations, and functions by interpreting representations of functions of two variables using symbolic algebra to explain mathematical relationships.

Objective
Given a set of formulas, the student will be able to use the GRAPH Menu and G-Solve Function to solve problems involving the investment of money.

Getting Started
Discuss the importance of saving money and the difference between simple and compound interest.

Prior to using this activity:
- Students should have a working knowledge of using the calculator to enter various formulas and display the corresponding graph.
- Students should be able to use the G-Solve Function to find specific x- and y-values.

Ways students can provide evidence of learning:
- The students will be able to discuss the results of the activity and justify their answers to specific questions.
- The students will be able to discuss how the graphical results correlate to their answers.

Common calculator or content errors students might make:
- When there are multiple formulas used, students could utilize the wrong formula or substitute the incorrect information for a particular variable.

Definitions
- Interest & Compound Interest
- Principal
- Rate of Interest
- Future value

Formulas

Future Value of an Annuity: $A = \dfrac{P}{i}\left[(1+i)^n - 1\right]$

[Note: Variables defined in Activity]

Number of Payments: $N = \dfrac{\log(1+iF \div P)}{\log(1+i)}$

Monthly Payment for a Desired Future Value: $P = \dfrac{iF}{(1+i)^n - 1}$

Saving for a Rainy Day

Saving for a Rainy Day   "How To"

The following will demonstrate how to enter a given formula into the GRAPH module the Casio *fx-9750GII*, graph the data, and use G-Solve to find x- and y-values.

Example Formulas:

$$A = P\left(\frac{1+r}{n}\right)^n \quad \text{where } n = 4, P = 100, \text{ and } r = x$$

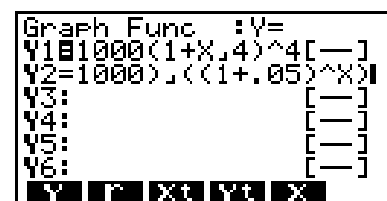$$A = \frac{nP}{(1+r)^n} \qquad \text{where } n = x, P = 1000, \text{ and } r = 0.05$$

Steps for using the GRAPH Menu:

1. From the Main Menu, press [3] for the GRAPH icon.

2. Enter the first formula into **Y1**: by entering:

    [1] [0] [0] [0] [(] [1] [+] [X,θ,T] [a b/c] [4] [)] [∧] [4] [EXE] .

3. Enter the second formula into **Y2**: by inputting:

    [(] [X,θ,T] [×] [1] [0] [0] [0] [)] [÷] [(] [(]

    [1] [+] [.] [0] [5] [)] [∧] [X,θ,T] [)] [EXE] .

To select the viewing window for the graph of this data:

1. Press the (▲) once, then press [F1] to deselect **Y2**:

2. Press [F6] to display the graph of **Y1**:.

3. Press [SHIFT] [F2] (V-Window), then [F5] (AUTO).

4. To see only the first quadrant, press [SHIFT] [F3] [0] [EXE]

    (▼) three times, [0] then [EXE] three times.
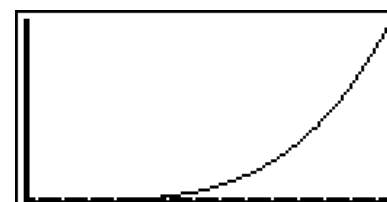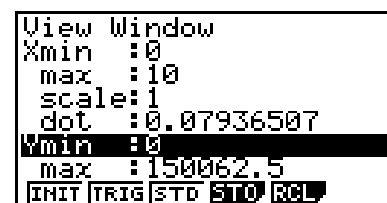
Steps for using G-Solve:

1. Press SHIFT F5 (G-Solv), F6 (▷)and F1 (Y-CAL).

2. To find the y-value when x = 0.1, input the following: ⋅ 1 EXE .

3. Press SHIFT F5 (G-Solv), F6 (▷) and F2 (X-CAL).

4. To find the x-value when y = 2000, input the following: 2 0 0 0 EXE .

Saving for a Rainy Day   Activity
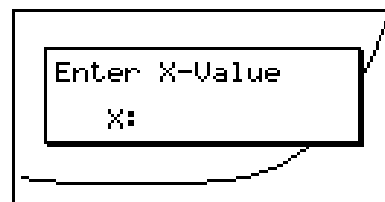
Investing for the future is usually the last thing on a person's mind when they are just entering the workforce. Paying bills, buying groceries, and purchasing a home are usually at the top of the list. However, putting money away in some form of savings should be the number one priority of every budget. Social security and retirement plans are often not enough to allow a person to continue to afford their current lifestyle.

In this activity, you will investigate how much a single investment will earn, calculate the balance of an account with a given monthly payment, and determine the investment amount that is needed to reach a specific financial goal.

Questions

The future value of an annuity can be calculated using the following formula:

$$A = \frac{P}{i}\left[(1+i)^n - 1\right]$$

Where *A* is the ending balance, *P* is the principal, *i* is the rate of interest, and *n* is the number of times the interest is calculated.

1.   What would be the future value of an annuity if $1000 is invested yearly for 5 years at 2.5% APR?

     _____

2.   What would be the future value of an annuity if $1000 is invested yearly for 5 years at 4% APR?

     _____

3.   What would be the future value of an annuity if $1000 is invested yearly for 5 years at 10% APR?

     _____

4.   What would be the future value of an annuity if $10,000 is invested yearly for 5 years at 2.5% APR?

     _____

5. What would be the future value of an annuity if $10,000 is invested yearly for 5 years at 4% APR?

_____

6. What would be the future value of an annuity if $10,000 is invested yearly for 5 years at 10% APR?

_____

7. For a principal amount of $10,000, what is the difference between the amount of income earned at 2.5% and the amount of income earned at 10%?

_____

8. How long would it take for an investment of $500, earning 4% APR, compounded annually, to earn $1,000?

_____

9. How long would it take for an investment of $500, earning 4% APR, compounded annually, to earn $2,000?

_____

10. How long would it take for an investment of $500, earning 4% APR, compounded annually, to earn $3,000?

_____

11. How long would take to earn $5,000?

_____

The formula for finding the number of payments, at a given percent, for a particular annual investment, to reach a specified goal is:

$$N = \frac{\log(1 + iF \div P)}{\log(1 + i)}$$

Where *N* is the number of payments, *i* is the interest rate, *F* is the future value of the investment, and *P* is the monthly amount invested.

12. Calculate the future value of an investment of $100 a year, at 5% APR, for 10 years.

   _____

13. Calculate the future value of an investment of $100 a year, at 5% APR, for 30 years.

   _____

14. Calculate the difference between question 12 and question 13.

   _____

15. Calculate the future value of an investment of $500 a month, at 5% APR, for 10 years.

   _____

16. Calculate the future value of an investment of $500 a month, at 5% APR, for 30 years.

   _____

17. Calculate the difference between question 15 and question 16.

   _____

The smart move is to start investing early and put aside a set amount each year. The formula for finding the amount of money earned from an annual investment at a given rate is:

$$P = \frac{iF}{(1+i)^n - 1}$$

Where P is the annual amount invested, i is the rate of interest, n is the number of times the interest is calculated, and F is the future value of the investment.

18. Calculate the annual investment needed at 10% APR to earn $100,000 in 10 years.

   _____

19. What is the difference between the investment for 10 years and the investment at 30 years?

_____

20. What is the benefit of starting to invest early?

_____

_____

Extension

1. Credit cards charge interest, however, that interest is compounded daily. What changes would need to be made to the compound interest formula to be able to calculate credit card interest that is compounded daily? Explain your thinking.
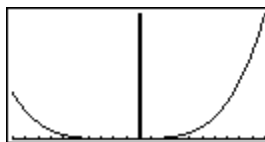
_____

_____

_____

_____

_____

Solutions

1. $5256.33

2. $5416.32

3. $6105.10

```
Graph Func  :Y=
Y1=(1000÷X)((1+X)^5-1
Y2:               [—]
Y3:               [—]
Y4:               [—]
Y5:               [—]
Y6:               [—]
 Y   r  Xt  Yt  X
```

4. $52,563.28

5. $54,163.22

6. $61,051.00

```
Graph Func  :Y=
Y1=(10000÷X)((1+X)^5-
Y2:               [—]
Y3:               [—]
Y4:               [—]
Y5:               [—]
Y6:               [—]
 Y   r  Xt  Yt  X
```
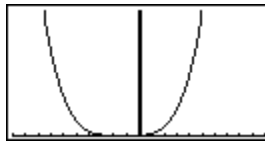
7. $8,487.72

8. 1.9 years

9. 3.8 years

10. 5.5 years

```
Graph Func  :Y=
Y1=(500÷.04)((1.04^X)
Y2:               [—]
Y3:               [—]
Y4:               [—]
Y5:               [—]
Y6:               [—]
 Y   r  Xt  Yt  X
```

11. Answers will depend on student experience
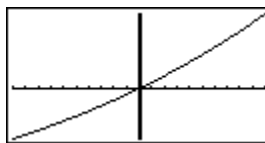
12. $1,257.79

13. $6,643.88

14. $5,386.09

```
Graph Func  :Y=
Y1=(log (1+(.05X÷100)
Y2:               [—]
Y3:               [—]
Y4:               [—]
Y5:               [—]
Y6:               [—]
 Y   r  Xt  Yt  X
```
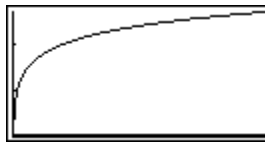
15. $6,288.95

16. $33,219.42

17. $26,930.47

```
Graph Func  :Y=
Y1=(log (1+(.05X÷500)
Y2:               [—]
Y3:               [—]
Y4:               [—]
Y5:               [—]
Y6:               [—]
 Y   r  Xt  Yt  X
```
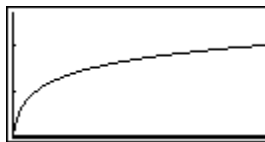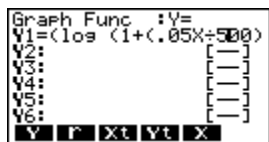
18. $627.45

19. $566.66

20. Answers will vary depending on student experience but should involve the fact that the longer money is invested, the more money you will have.

Extension Solution

1. Answers will vary. Things of note would be the interest rate is being charged not received and that *n* would be changed to a compounded daily rate.

Quite the Quadrilateral   Teacher Notes

Topic Area: Properties of Parallelograms

NCTM Standards:
- Use geometric ideas to solve problems in, and gain insights into, other disciplines and other areas of interest such as art and architecture.
- Use Cartesian coordinates and other coordinate systems, such as navigational, polar, or spherical systems, to analyze geometric situations.
- Investigate conjectures and solve problems involving two- and three-dimensional objects represented with Cartesian coordinates.

Objective
The student will be able to use algebra and statistics to prove that a quadrilateral is a parallelogram, demonstrate that the opposite sides are equal, demonstrate that the diagonals bisect each other, and prove that the opposite angles are equal.

Getting Started
As a class, review the meaning of slope and the slope-intercept form of an equation; include in the discussion the relationship of the slopes between parallel lines and perpendicular lines. Review methods of proving triangles congruent using the Side-Side-Side method.

Prior to using this activity:
- Students should be able to find the xy-line for a pair of coordinates using a graphing calculator.
- Students should be able to perform calculations involving square roots, ratios, and parentheses using a graphing calculator.
- Students should know the formula for finding the distance between two points.

Ways students can provide evidence of learning:
- The student will be able to write conjectures pertaining to a parallelogram.
- The student will be able apply the properties of a parallelogram to real-life problems.

Common mistakes to be on the lookout for:
- Students may confuse the x and y values in the calculations.
- Students may enter the problem incorrectly into the calculator.

Definitions
- Parallelogram
- Perpendicular
- Endpoint
- Slope
- Diagonal
- Intersection
- Midpoint
- Congruent
- Hypotenuse
- Leg

Quite the Quadrilateral

"How-To"

The following will demonstrate how to enter a set of coordinates into two lists using the Statistics mode of the Casio *fx-9750GII*. After the list is set up, you will find the slope of a line containing the points, save the equation in the Graph mode, and find the intersection of two lines. You will then find the length of a segment.

Line segment AB has endpoints at (-5, -2) and (3, 6) and segment CD has endpoints at (-6, 4) and (3, -7). Find the slope for each line segment, the coordinates of their intersection, and the length of $\overline{AB}$.

To enter values into a list and find the line of best fit:

1.   From the Main Menu, highlight the Statistics icon
       and press EXE or press 2 .

2.   To label the first column, highlight the space below
       List 1 and press ALPHA ➕ (X) EXE .

3.   To label the second column, highlight the space
       below **List 2** and press ALPHA ➖ (Y) EXE .

4.   Enter the x-values into List 1 and the y-values into
       List 2. Be sure to press EXE after each value.

5.   To view the points, press F1 (GRPH) F6 (Set)
       ⬇ F1 (GPH1) ⬇ F2 (XY) ⬇ F1 (List) 1 ⬇
       F1 (List) 2 EXE F1 (GPH1).

6.   Press EXIT and F1 (GPH1) to view the graph.

7.   Press F1 (Calc) F2 (X) F1 (ax+b) to find the line
       of best fit.

8.   Press F5 (Copy) EXE to copy the equation into
       the graph function.

9.   Repeat the same steps to find the equation for the second segment.
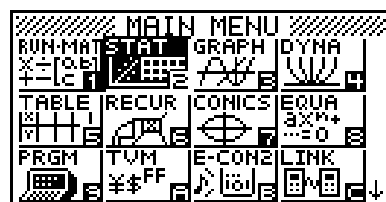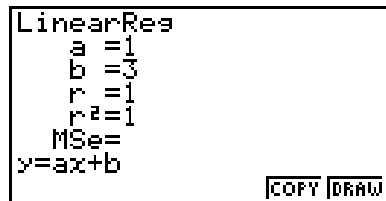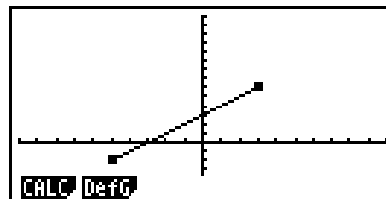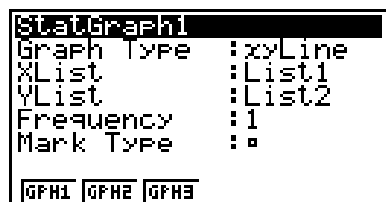
To graph the two equations and find the intersection:

1. From the Main Menu, highlight the Graph icon and press [EXE] or press [3].

2. To graph the two equations, highlight each equation and press [F1] (Sel) to turn the function on; when the equal signs are highlighted, you know the equation is selected. Then press [F6] (Draw).

3. While viewing the graph, press [F5] (G-Solv) [F5] (ISCT) to find the intersection of the two equations.

4. The coordinates are displayed at the bottom of the screen.
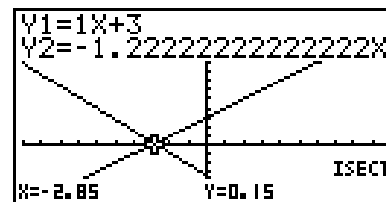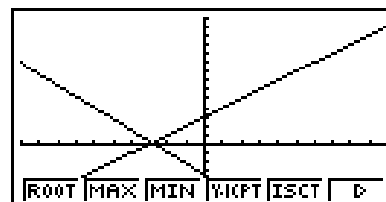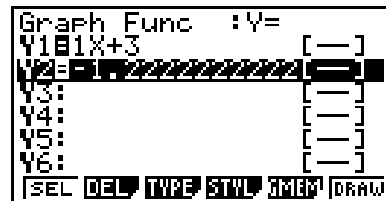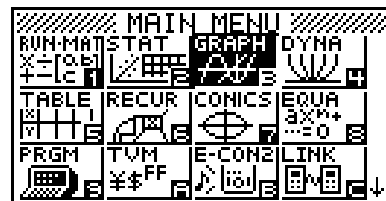
To find the length of $\overline{AB}$:

1. Using the distance formula,

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$, press [SHIFT] [$x^2$] [(] [(] [3] [−] [(−)] [5] [)] [x^2] [+] [(] [6] [−] [(−)] [2] [)] [x^2] [)] [EXE] to find the length of $\overline{AB}$.
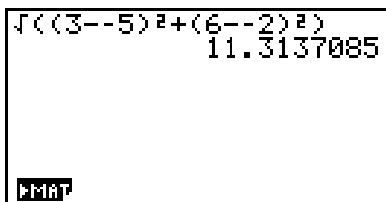
Quite the Quadrilateral   Activity

The parallelogram is a special quadrilateral with special properties that is used in a variety of areas, especially in design. In this activity, we will explore the properties and then solve some problems using those properties.

Questions

The diagram at the right shows Quad ABCD. By definition, a parallelogram is a quadrilateral with both pairs of opposite sides parallel.
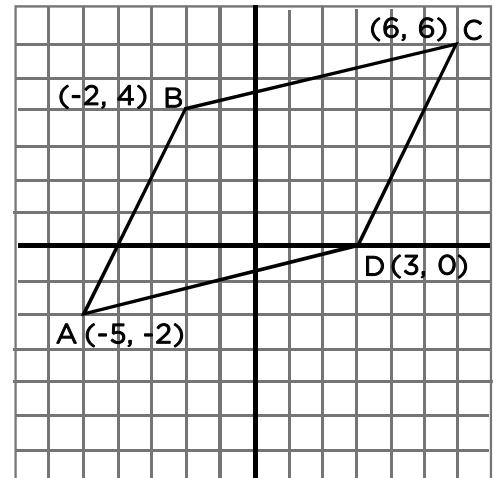
1. Find the equation of a line that contains the following points:
    a.   points B and C
    _____

    b.   points A and D
    _____

2. What is the slope for each line?
    _____

3. Find the equation of a line that contains the following points:
    a.   points A and B
    _____

    b.   points D and C
    _____

4. What is the slope for each line?
    _____

5. Are the opposite sides parallel?
    _____

Let us see what else we can find out about the sides of a parallelogram.

6. Find the length for the following segments to the nearest tenth.
    a. $\overline{BC}$: _____
    b. $\overline{AD}$: _____

7. Find the length for the following segments to the nearest tenth.

 a. $\overline{AB}$: _____

 b. $\overline{DC}$: _____

8. What can you conclude about the opposite of a parallelogram?

 _____

Draw the two diagonals for the figure. We are now going to look at their properties in relation to quadrilaterals.

9. Find the equation for the following segments.

 a. $\overline{AC}$: _____

 b. $\overline{BD}$: _____

10. Find the coordinates for the intersection of the two diagonals. Draw it on the diagram and label it E.

 _____

11. Find the length of the following segments to the nearest tenth.

 a. $\overline{AE}$: _____

 b. $\overline{CE}$: _____

12. Find the length of the following segments to the nearest tenth.

 a. $\overline{BE}$: _____

 b. $\overline{DE}$: _____

13. What conclusion can be made about the diagonals of a parallelogram?

 _____

14. Using the information above, determine the reason why each of the following pairs of triangles are congruent.

 a. $\triangle ABC \cong \triangle ADC$ by _____

 b. $\triangle ABD \cong \triangle CDB$ by _____

15. Since the two pairs of triangles are congruent, then give two pairs of angles that are equal.

 _____ and _____

16. What conclusion can be made about angles in a parallelogram?

_____

One method of demonstrating vector addition is by creating a parallelogram. The sum is the coordinates of the fourth vertex of the parallelogram.

17. Given the diagram below and using the properties of parallelograms, find the sum of v_1 and v_2.

_____

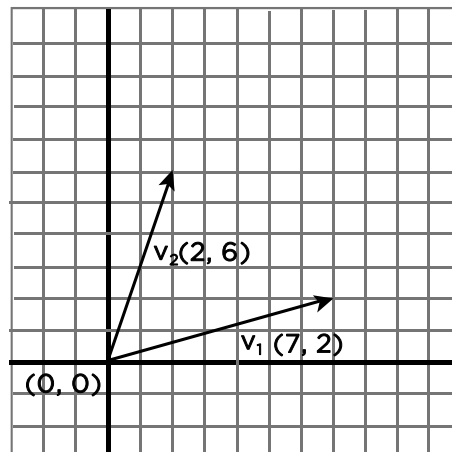18. The magnitude of a vector is equal to its length. Find the magnitudes of v_1, v_2, and the resulting vector to the nearest hundredth.
    a. v_1

_____

    b. v_2

_____

    c. $v_1 + v_2$

_____

$v_2(2, 6)$

$v_1(7, 2)$

$(0, 0)$

Solutions:

1. a. y = 0.25x + 4.5

```
LinearReg(ax+b)
   a =0.25
   b =4.5
   r =1
   r²=1
  MSe=
y=ax+b
            COPY DRAW
```

   b. y = 0.25x − 0.75

```
LinearReg(ax+b)
   a =0.25
   b =-0.75
   r =1
   r²=1
  MSe=
y=ax+b
            COPY DRAW
```

2. The slope of each line is 0.25.

3. a. y = 2x + 8;

```
LinearReg(ax+b)
   a =2
   b =8
   r =1
   r²=1
  MSe=
y=ax+b
            COPY DRAW
```

   b. y = 2x − 6

```
LinearReg(ax+b)
   a =2
   b =-6
   r =1
   r²=1
  MSe=
y=ax+b
            COPY DRAW
```

4. The slope of each line is 2.

5. The opposite sides are parallel.

6. a. $\sqrt{(6--2)^2 + (6-4)^2}$ = 8.2

   b. $\sqrt{(3--5)^2 + (0--2)^2}$ = 8.2

7. a. $\sqrt{(-2--5)^2 + (4--2)^2}$ = 6.7

   b. $\sqrt{(6-3)^2 + (6-0)^2}$ = 6.7

8. Opposite sides of the parallelogram are equal.

9. a. y = 0.36x + 3.81


```
LinearReg(ax+b)
    a =0.36363636
    b =3.81818181
    r =1
    r²=1
   MSe=
y=ax+b
                  COPY DRAW
```

b. y = −0.8x + 2.4

```
LinearReg(ax+b)
    a =-0.8
    b =2.4
    r =-1
    r²=1
   MSe=
y=ax+b
                  COPY DRAW
```

10. (0.5, 2)

```
Y1=0.72727272727272727X
Y2=-0.8X+2.4


                  ISECT
X=0.5        Y=2
```
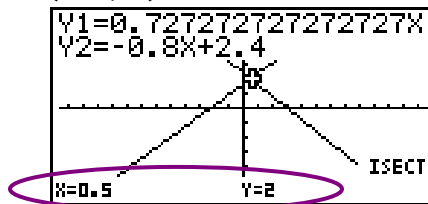
11. a. $\sqrt{(0.5 - -5)^2 + (2 - -2)^2} = 6.8$

b. $\sqrt{(0.5 - 6)^2 + (2 - 6)^2} = 6.8$

12. a. $\sqrt{(0.5 - -2)^2 + (2 - 4)^2} = 3.2$

b. $\sqrt{(0.5 - -5)^2 + (2 - 4)^2} = 3.2$

13. The diagonals bisect each other.

14. a. SSS Congruency
     b. SSS Congruency

15. $\angle ABC \cong \angle CDA$ & $\angle BAD \cong \angle DCB$

16 Opposite angles of a parallelogram are equal.

17. $v_1 + v_2 = \langle 9,8 \rangle$

18. a. $v_1 = \sqrt{2^2 + 6^2} = 6.32$ units

b. $v_2 = \sqrt{7^2 + 2^2} = 7.28$ units

c. $v_1 + v_2 = \sqrt{9^2 + 8^2} = 12.04$ units

v₁(2, 6)

v₂(7, 2)

(0, 0)

The Cost of Car Insurance   Teacher Notes
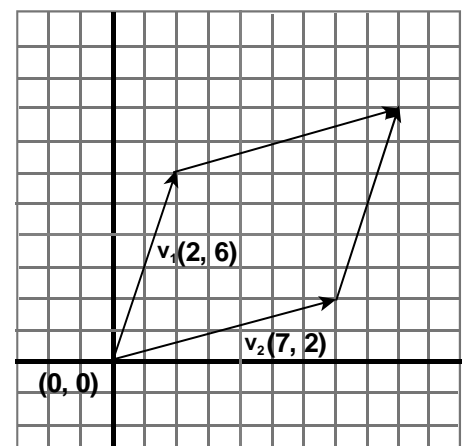
Topic: Data Analysis and Probability

NCTM Standard(s)
- For univariate measurement data, be able to display the distribution, describe its shape, and select and calculate summary statistics.

Objective:
Given a set of data, the student will be able to enter data into the statistics menu of the Casio *9750 GII*, graph the data using a median box-and-whisker graph, and calculate the measures of central tendency.

Getting Started
Have the students work in pairs or small groups and come up with examples of using one-variable data, what kind of information can be obtained from one-variable data and what types of graphs can be used to represent one-variable data.

Prior to using this activity:
- The student should be able to calculate basic statistics.
- The students should be familiar with interquartile values.

Ways students can provide evidence of learning:
- Given a set of data, the student should be able to create a box and whisker plot.
- The student should be able to answer questions about the range of a set of data.

Common mistakes to be on the lookout for:
- Students may pick a measure of central tendency that does not best describe the situation.
- Students may not understand the effect that outliers have on the set of data.

Definitions:
- Mean
- Median
- Mode
- Standard Deviation
- Interquartile Range
- Central tendency

# The Cost of Car Insurance   "How-To"

The following will demonstrate how to enter a set of data into the Casio *fx-9750GII*, graph the data using a Box and Whisker Plot and find important information from the graph.

Scores on the First Math Test									
55	60	75	80	90	65	75	60	50	80

Scores on the Second Math Test									
75	90	85	60	95	85	80	70	95	100

To enter the data from the table in the problem:

1.   From the Main Menu, highlight the STAT icon and press EXE or 2.

2.   To clear previous data lists press:
     F6 (▷) F4 (DEL-A) F1 (Yes).

3.   Enter the data by typing each number, pressing EXE after each entry.

4.   The display should look like the screen shot on the right when completed.
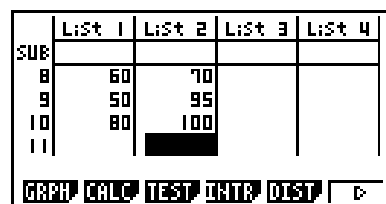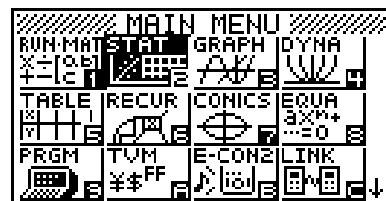
To select the type of graph for this data:

1.   Press F1 (GRPH) and F6 (SET) to set the type of graph for StatGraph1.

2.   Press ⬇ to highlight Graph Type.

3.   There are five choices: Scat, *XY*, NPP, Pie, and (▷). Selecting F6 (▷) will provide more graph choices.
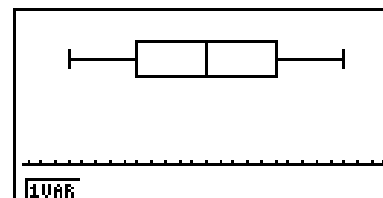
4.   Press $\boxed{\text{F2}}$ (Box) for a box-and-whisker plot.

```
StatGraph1
Graph Type   :MedBox
XList        :List1
Frequency    :1
Outliers     :Off

Hist Box Bar N·Dis Brkn  ▷
```
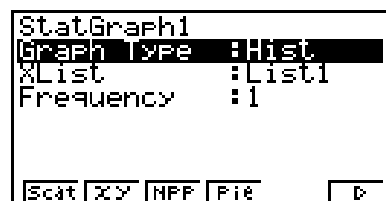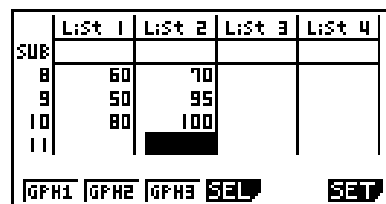
5.   Make sure that the XList is List 1 and a
        Frequency of 1. If not, scroll down and press
        $\boxed{\text{F1}}$ to select a frequency of 1.

```
StatGraph1
Graph Type   :Hist
XList        :List1
Frequency    :1

 1  LIST
```

6.   Press $\boxed{\text{EXIT}}$, then $\boxed{\text{F1}}$ (GPH1) to view the graph.

7.   Pressing $\boxed{\text{F1}}$ will display the statistical data
        from the list.

```
 ├──┤ ┌────┬──┐ ├──┤

1VAR
```

To graph multiple sets of data:

1.   Press $\boxed{\text{EXIT}}$ to go back one screen.

```
StatGraph2
Graph Type   :Scatter
XList        :List1
YList        :List3
Frequency    :1
Mark Type    :×

GPH1 GPH2 GPH3
```

2.   Press $\boxed{\text{F6}}$ (SET) and $\boxed{\text{F2}}$ (GPH2) to set the
        type of graph for StatGraph 2.

3.   Press $\boxed{\text{F2}}$ (Box) for a box-and-whisker plot,
        then press $\blacktriangledown$ to change the XList to List 2.

```
StatGraph2
Gl┌─────────────────┐
 X│ Select List No. │
 Fl                 │
 Ol    List[1~26]: 2 │
  └─────────────────┘
LIST
```

4.   Press $\boxed{\text{F4}}$ (SEL) to select the graphs to be
        displayed.

```
      List 1  List 2  List 3  List 4
SUB
   8      60      70
   9      50      95
  10      80     100
  11

GPH1 GPH2 GPH3 SEL       SET
```

5.   Arrow down to the graphs that you would
        like to see drawn and press $\boxed{\text{F1}}$ (On).
        Then, press $\boxed{\text{F6}}$ (DRAW).

```
StatGraph1   :DrawOn
StatGraph2   :DrawOff
StatGraph3   :DrawOff

On Off               DRAW
```

```
 ├──┌────┬──┐──┤
 ├──┌────┬──┐──┤

1VAR
```
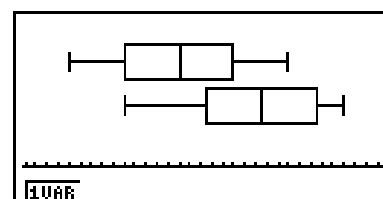
To perform a 2 variable statistic analysis of the data:

1.   [EXIT] twice until you are at the main stat screen.

2.   Press [F2] (CALC), then [F2] (2VAR) for a
         two-variable analysis.

3.   Scroll down to see the data.

The Cost of Car Insurance   Activity

For many years, actuaries have kept track of the driving records of car insurance policy holders. These statistics compare males and females and those under or above 21 years old. This data is used to determine the amount paid for car insurance premiums. In this activity, you will compare the cost of car insurance premiums that resulted from the analysis of this data.

Insurance Co.	Female < 21	Female ≥ 21	Male < 21	Male ≥ 21
Company A	$2,046	$1,520	$3,041	$2,108
Company B	$1,825	$1,239	$2,617	$1,514
Company C	$2,152	$1,637	$2,946	$1,701
Company D	$1,773	$1,129	$2,459	$1,477
Company E	$2,381	$1,748	$3,291	$2,439

Questions

1.   What is the range of costs for car insurance for a female under 21?
_____

      What is the mean cost?
_____

2.   What is the range of costs for car insurance for a male under 21?
_____

      What is the mean cost?
_____

3.   What is the difference between the mean costs of a female and male driver under 21?
_____

4.   Can you think of some reasons why the cost is so different for male and female drivers under the age of 21?
_____
_____
_____

5.   Use your Casio *9750GII* to graph a box and whisker for each of the age and gender groups. Draw a sketch of each graph. Be sure to label the interquartile values for each age and gender group.

Female < 21

Female ≥ 21

Male < 21

Male ≥ 21

6.   What is the range of costs for car insurance for females over 21 years old?
_____

What is the mean cost?
_____

7.   What is the range of costs for car insurance for males over 21 years old?
_____

What is the mean cost?
_____

8.   What is the difference between a male driver over 21, and a female driver over 21 years old?
_____

_____

_____

9.   Looking at the data overall, compare the mean and median costs for all the data sets, and find the best insurance company and the yearly rate each would pay for the following:

Male, 17 years old   _____   Amount paid   _____

Female, 18 years old   _____   Amount paid   _____

Male, 76 years old   _____   Amount paid   _____

Female, 35 years old   _____   Amount paid   _____

10.   Do you think that this is fair? Why or why not?

_____

_____

_____

Extensions

Speak with some of your local insurance companies and get rate quotes for someone your age, but for different types of vehicles, like a sports car, a truck, a compact car or an old Caprice.

Solutions

1.   Range = $2381 - $1773 = $608

```
1-Variable
n    =5              ↑
minX =1773
Q1   =1799
Med  =2046
Q3   =2266.5
maxX =2381
               DRAW
```
```
2381-1773
                608

▶MAT
```

        Mean = $2035.40

```
1-Variable
x̄    =2035.4
Σx   =10177
Σx²  =2.096E+07
σx   =221.932061
sx   =248.127588
n    =5
               DRAW
```

2.   Range = $3291 - $2459 = $8732

```
1-Variable
n    =5              ↑
minX =2459
Q1   =2538
Med  =2946
Q3   =3166
maxX =3291
               DRAW
```
```
3291-2459
                832

▶MAT
```

        Mean = $2870.80

```
1-Variable
x̄    =2870.8
Σx   =14354
Σx²  =4.1652E+07
σx   =298.390616
sx   =333.610851
n    =5
               DRAW
```

3. $2870.80 - $2035.40 = $835.40

```
2870.8-2035.4
                835.4



▶MAT
```

4.   Answers will vary
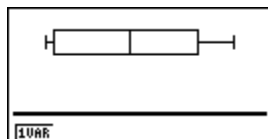
5.   Female < 21   Female ≥ 21   Male < 21   Male ≥ 21
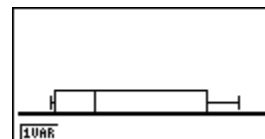
```
1-Variable
n    =5          ↑
minX =1773
Q1   =1799
Med  =2046
Q3   =2266.5
maxX =2381
            DRAW
```
```
1-Variable
n    =5          ↑
minX =1129
Q1   =1184
Med  =1520
Q3   =1692.5
maxX =1748
            DRAW
```
```
1-Variable
n    =5          ↑
minX =2459
Q1   =2538
Med  =2946
Q3   =3166
maxX =3291
            DRAW
```
```
1-Variable
n    =5          ↑
minX =1477
Q1   =1495.5
Med  =1701
Q3   =2273.5
maxX =2439
            DRAW
```

6. Range = $1748 - $1129 = $619

    ```
    1-Variable
    n       =5
    minX =1129
    Q1      =1184
    Med   =1520
    Q3      =1692.5
    maxX =1748
                    DRAW
    ```
    ```
    1748-1129
                    619

    ▶MAT
    ```

    Mean = $1456.60

    ```
    1-Variable
    x̄       =1454.6
    Σx      =7273
    Σx²    =1.0855ε+07
    σx      =235.001787
    sx      =262.739985
    n       =5
                    DRAW
    ```

7. Range = $2439 - $1477 = $962

    ```
    1-Variable
    n       =5
    minX =1477
    Q1      =1495.5
    Med   =1701
    Q3      =2223.5
    maxX =2439
                    DRAW
    ```
    ```
    2439-1477
                    962

    ▶MAT
    ```

    Mean = $1847.80

    ```
    1-Variable
    x̄       =1847.8
    Σx      =9239
    Σx²    =1.7759ε+07
    σx      =370.860297
    sx      =414.634417
    n       =5
                    DRAW
    ```
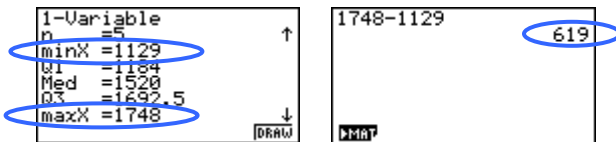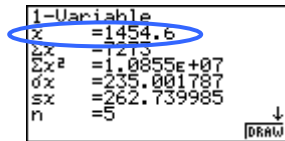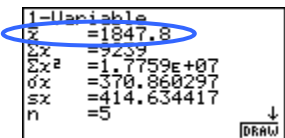
8. $1847.80 – $1456.60 = $391.20

    ```
    1847.8-1456.6
                    391.2

    ▶MAT
    ```
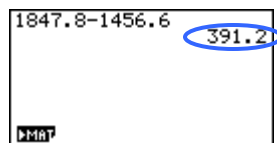
9. Male, 17 years old: Company D   Amount paid: $2,459
    Female, 18 years old: Company D   Amount paid: $1,773
    Male, 76 years old: Company D   Amount paid: $1,477
    Female, 35 years old: Company D   Amount paid: $1,129

10. Answers will vary

Area of a Triangle and Circular Sector   Teacher Notes

Topic Area: Trigonometric Applications

NCTM standards:
- Develop fluency in operations with real numbers using technology for more-complicated cases.
- Understand functions by interpreting representations of functions.

Objective

To calculate the area of a triangle and a circular sector using trigonometry.

Getting Started

In this activity, the student will learn how to calculate the area of a triangle and a circular sector using trigonometry. The area of a triangle is defined to be one-half the product of the lengths of the two sides (*a, b*) and the sine of angle (*C*) included between those two sides. The formula looks like:

$$\text{area} = \frac{1}{2} \cdot a \cdot b \cdot \sin C$$
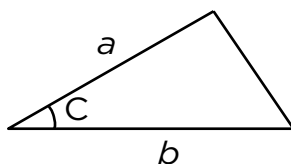
Another useful formula for finding the area of a triangle is Heron's formula In this formula the students need to know the lengths of the three sides (*a, b, c*) to find the area of the triangle. Heron's formula is:

$$\text{area} = \sqrt{s(s-a)(s-b)(s-c)} \quad \text{where } s = \frac{1}{2}(a + b + c)$$
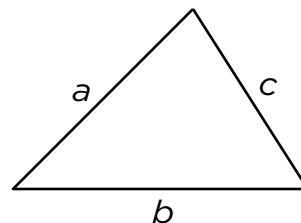
The area of a circular sector is defined to be one-half of the product of the radius (*r*) squared and the central angle. The formula is:
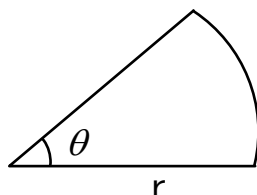
$$\text{area} = \frac{1}{2} \cdot r^2 \cdot \theta \qquad \text{or}$$
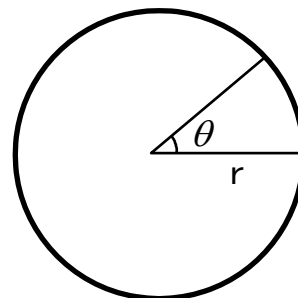
Prior to using this activity:

- Students should understand the difference between right triangles and non-right triangles.
- Students should know the difference between degrees and radians as angle measurements.

Ways students can provide evidence of learning:

- Students will be able to calculate the area of any triangle.
- Students will be able to calculate the area of a circular sector.

Common mistakes to be on the lookout for:

- Students may forget to switch between degree and radian mode on the calculator.
- When using Heron's formula, students may incorrectly calculate s (or not calculate it at all).

Definitions

- arc
- sector
- circular sector
- degree
- radian

Area of a Triangle and Circular Sector   "How-To"

The following will demonstrate how to enter the data into the Casio *fx-9750GII* and interpret the results.

To set the calculator to calculate in degrees:

1. From the Main Menu, highlight the RUN•MAT icon and press [EXE] or [1].

2. Make sure the calculator is in degree mode by pressing [SHIFT] [MENU], move the cursor down to Angle. Press [F1] (Deg) to change it into degrees, then [EXIT].

```
Mode         :Comp
Frac Result :d/c
Func Type    :Y=
Draw Type    :Connect
Derivative   :Off
Angle        :Rad
Complex Mode:Real      ↓
 Deg  Rad  Gra
```

```
Mode         :Comp
Frac Result :d/c
Func Type    :Y=
Draw Type    :Connect
Derivative   :Off
Angle        :Deg
Complex Mode:Real      ↓
 Deg  Rad  Gra
```

To solve equations including trigonometric functions:

For this example, use the equation for area = $\frac{1}{2}a \bullet b \bullet \sin C$.

Let *a* = 2, *b* = 4, and *C* = 30°

```
.5×2×4×sin 30
                    2



 ►MAT
```

1. To calculate the area with the given parameters, press: [•] [5] [×] [2] [×] [4] [×] [sin] [3] [0] [EXE].
    The area is 2.

To use the Solver feature:

For this example, we will use the equation for kinetic energy, KE = 0.5 mv^2.

Let m = 10 kg, v = 25 $\frac{m}{sec^2}$.

1. From the Main Menu, highlight the EQUA icon and press [EXE] or [8], then press [F3] (SOLV).

2. To input the equation, press:
    [ALPHA] [•] [SHIFT] [•] [•] [5] [ALPHA] [7] [ALPHA] [2] [x^2] [EXE]
    Note: *k* is used to represent KE.

```
Equation

Select Type
F1:Simultaneous
F2:Polynomial
F3:Solver
 SIML  POLY  SOLV
```

```
Eq:K=.5MV²
     K=0
     M=0
     V=0
  Lower=-9E+99
  Upper=9E+99

 RCL  DEL         SOLV
```

3. Enter 10 for M and 25 for V, and then highlight K.

4. Press F6 (SOLV).

6. Kinetic energy for this example is 3125 kg• $\dfrac{m}{\sec^2}$.

```
Eq:K=.5MV²
   K=3125
Lft=3125
Rgt=3125


REPT
```

7. To use the Solver to find m , input values for K and V, highlight M and press F6 to find the value for M. Use this same method to find V.

Area of a Triangle and Circular Sector   Activity

Introduction

In this activity, you will learn how to calculate the area of a triangle and a circular sector using trigonometry. The area of a triangle is defined to be one-half of the product of the lengths of the two sides (*a, b*) and the sine of angle (*C*) included between those two sides. The formula looks like:

area = $\frac{1}{2}$•*a*•*b*•sin C
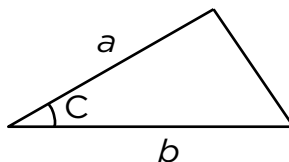
Another useful formula for finding the area of a triangle is Heron's formula In this formula you need to know the length of the three sides (*a, b, c*) to find the area of the triangle. Heron's formula is:

area = $\sqrt{s(s-a)(s-b)(s-c)}$ where s = $\frac{1}{2}$ (a + b + c)
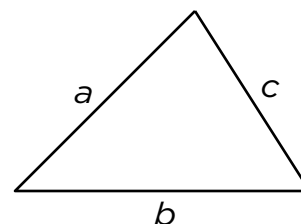
The area of a circular sector is defined to be one-half of the product of the radius ® squared and the central angle. The formula is:

area = $\frac{1}{2}$•r^2•θ or
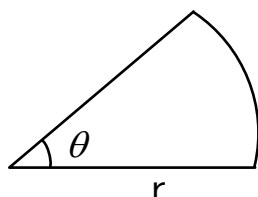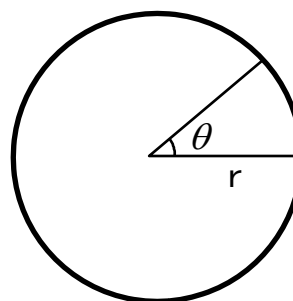
Questions

1.   Find the area of the following triangles for the given values using one of the following formulas:

area = $\frac{1}{2}$•*a*•*b*•sin C   area = $\frac{1}{2}$•*a*•*c*•sin B   area = $\frac{1}{2}$•*c*•*b*•sin A

a. *a* = 5, *b* = 7, *C* = 35°   area = _____

b. *b* = 10, *c* = 8, *A* = 28°   area = _____

c. *a* = 16, *c* = 4, *B* = 40°   area = _____

2. Find the area of the following triangles for the given values using Heron's formula and the Solve feature.

  a. $a = 5$, $b = 7$, $c = 10$ area = _____

  b. $a = 10$, $b = 8$, $c = 6$ area = _____

  c. $a = 6$, $b = 4$, $c = 8$ area = _____

3. Find the area of the following circular sectors for the given values using the formula in the Solve feature.

  a. $r = 8$, $\theta = \dfrac{\pi}{5}$ area = _____

  b. $r = 10$, $\theta = \dfrac{2\pi}{3}$ area = _____

  c. $r = 7$, $\theta = \dfrac{4\pi}{9}$ area = _____

Solutions

1. a. area = 10.0376 units^2
    b. area = 18.7789 units^2
    c. area = 7.7135 units^2

2. a. area = 16.2481 units^2

    b. area = 24 units^2

    c. area = 11.6190 units^2

3. *Note that angles are given in radians for this question. Calculator must be changed over to radian mode before proceeding.*

    a. area = 20.1062 units^2

    b. area = 104.7198 units^2

    c. area = 34.2085 units^2

A Graphical Look At Continuity   Teacher Notes

Topic: Continuity

NCTM Standard
- Organize and consolidate their mathematical thinking through communication; communicate their mathematical thinking coherently and clearly to peers, teacher, and others.

Objectives
The student will be able to develop a visual understanding of how limits and continuity relate and be able to understand and communicate what it means for a function to be continuous at a point.

Getting Started
This activity will have students explore the concept of continuity at a point. It will also allow them to discover that simply having a limit at a point will not guarantee that the function is also continuous. It also explores the idea that having a limit is a necessary, but not a sufficient condition to determine the continuity of a function at a point, and through all points.

Prior to using this activity:
- Students should be able to produce and manipulate graphs of functions manually and a graphing calculator.
- Students should be able to produce split-defined (or piecewise) functions.
- Students should have a basic understanding of the language of limits.

Ways students can provide evidence of learning:
- Students should be able to produce graphs of functions and communicate symbolically, graphically and verbally the relationship between having a limit and being continuous.

Common mistakes to be on the lookout for:
- Students may produce a graph on the calculator and not be able to communicate the concept of a split-defined function because the window chosen may produce the appearance of a single unbroken formula.
- Students may confuse the pixel values with the actual function values.

Definitions:
- Asymptote
- Continuity
- Discontinuity
- Limit
- Parabola
- Vertex

# A Graphical Look At Continuity "How-To"

The following will demonstrate how to graph a function, graph a split-defined function and examine its behavior on the CASIO *fx-9750GII*.

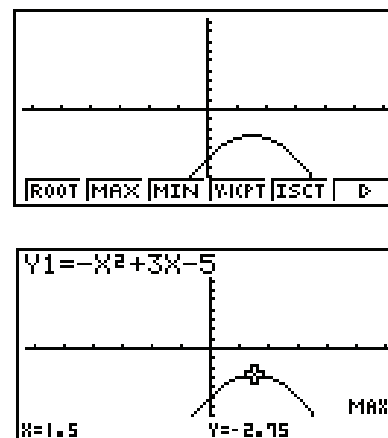Explore the behavior of the function f(x) = –x^2 + 3x – 5.

To display a graph of the function:

1. From the Main Menu, highlight the GRAPH icon and press EXE or press 3.

2. To delete any previous equations, highlight the equation and press F2 (DEL) F1 (Yes.)

3. Enter the equation in Y1 by pressing (–) X,θ,T x^2 + 3 X,θ,T – 5 EXE.

4. Set the view window to the initial screen by pressing SHIFT F3 (V-Window) F1 (INIT). Then, change the Ymin to –9.3 and the Ymax to 9.3. This view window will be used in this activity to give the best display of the quadratic functions.

5. Press EXIT to return to the main GRAPH screen.

6. Press F6 (DRAW) to view the graph of the function.
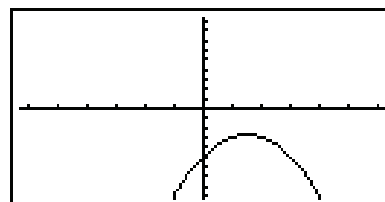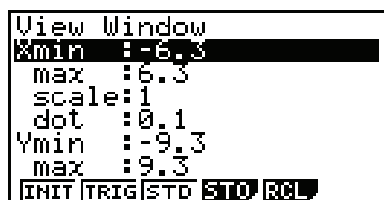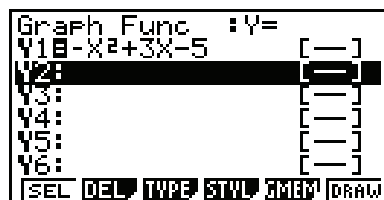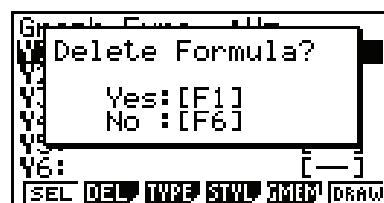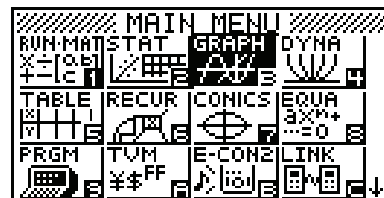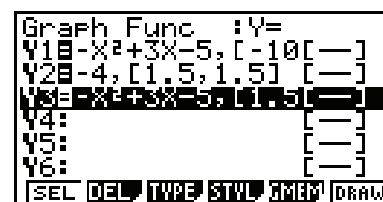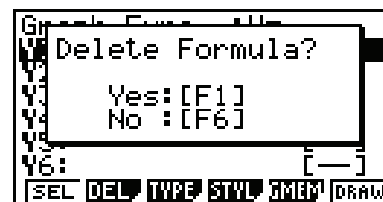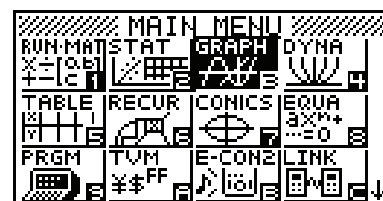
To find the vertex of the graph:

1.   Press SHIFT F5 (G-Solv) F2 (MAX). The coordinates of the vertex will be displayed at the bottom of the screen. [Note: F2 (MAX) was pressed since the vertex of this parabola is the highest, or maximum, point. If the graph of the parabola opened up, the vertex would be the lowest, or minimum, point and you would have chosen F3 (MIN).]

Explore the behavior of the split-defined function g(x) = $\begin{cases} -x^2 + 3x - 5, x < 1.5 \\ -4, x = 1.5 \\ -x^2 + 3x - 5, x > 1.5 \end{cases}$.

To graph a split-defined function:

1.   From the Main Menu, highlight the GRAPH icon and press EXE or press 3 .

2.   To delete any previous equations, highlight the equation and press F2 (DEL) F1 (Yes.)

3.   Enter each branch of the split-defined (piece-wise) function in its own Y= slot, then create the restrictions by putting the lower and upper bounds in brackets.
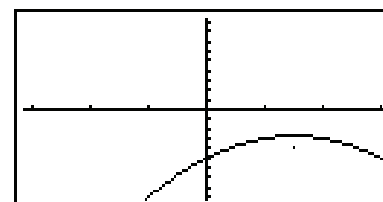   For Y1, press (−) X,θ,T x² + 3 X,θ,T − 5 , SHIFT + (−) 1 0 , 1 . 5 SHIFT − EXE .
    For Y2, press (−) 4 , SHIFT + 1 . 5 , 1 . 5 SHIFT − EXE .
    For Y3, press (−) X,θ,T x² + 3 X,θ,T − 5 , SHIFT + 1 . 5 , 1 0 SHIFT − EXE .

4.   Press F6 (DRAW) to view the graph of the functions.

A Graphical Look At Continuity   Activity

This activity will have you explore the concept of continuity at a point. It will also allow you to discover that simply having a limit at a point will not guarantee that the function is also continuous. Using the CASIO *fx-9750GII*, you will be working in pairs or small groups.

Questions

Explore the behavior of the function $f(x) = x^2 - x - 6$ around the vertex.

1. Graph the function using the initial view window, changing Ymin to –9.3 and Ymax to 9.3, and copy on the axes below.
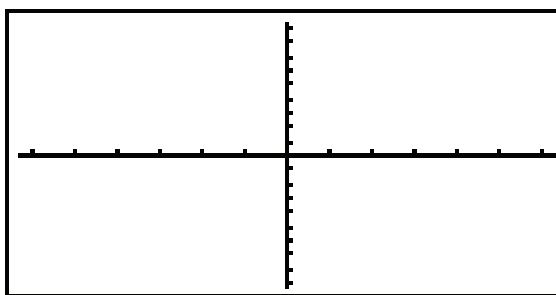
2. Find and record the vertex of the function.

    _____

3. Trace to the vertex and zoom in, record what you see.

    _____

    _____

    _____

4. What does the value of $\lim\limits_{x \to 0.5} f(x)$ appear to be? Explain how you arrived at that answer.

    _____

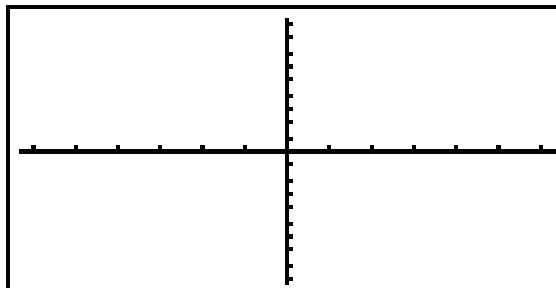   _____

    _____

5. Explore the behavior of the following split-defined function:

$$g(x) = \begin{cases} x^2 - x - 6, & x < 0.5 \\ -6, & x = 0.5 \\ x^2 - x - 6, & x > 0.5 \end{cases}$$

Use the same viewing window as before. Record what you see below.

6. What does the value of $\lim_{x \to 0.5} g(x)$ appear to be?

_____

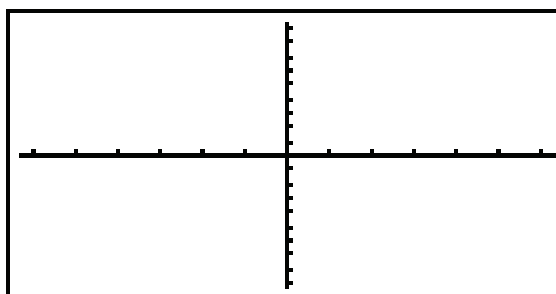7. How does it compare to the $\lim_{x \to 0.5} f(x)$?

_____

_____

8. Now trace to a value where x = 0.5 and zoom in. Describe and record what you see.

_____

_____

_____

9.   Find the following limits:

    a. $\lim\limits_{x \to 0.5^+} g(x)$ _____

    b. $\lim\limits_{x \to 0.5^-} g(x)$ _____

    c. $\lim\limits_{x \to 0.5} g(x)$ _____

10.   Find g(0.5). How does this compare to your answer for $\lim\limits_{x \to 0.5} g(x)$?

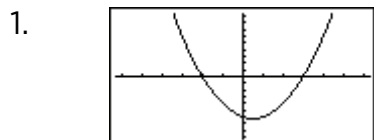_____

_____

_____

11.   Draw a conclusion about the relationship between limits and continuity.

_____

_____

_____
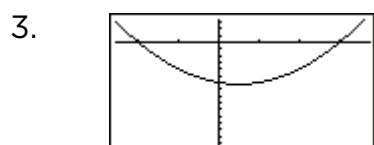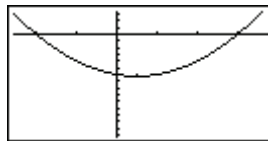
Solutions

1.

2.   (0.5, −6.25)

3.

Nothing unusual should be seen. The function is continuous.

4.   The limit is -6.25, the vertical value of the vertex. Answer will vary; care should be taken to point out that simply tracing to a value is not confirmation enough and can be tricky. Direct substitution is a valid explanation. A good answer might also include a mention of "passing through" or even a mention of continuity.

5.


```
Graph Func  :Y=
Y1◼X²−X−6,[−10,.[—]
Y2◼−6,[.5,.5]    [—]
Y3=X²−X−6,[.5,10]
Y4:              [—]
Y5:              [—]
Y6:              [—]
 Y  r  Xt  Yt  X
```


[Note: The discontinuity will not be immediately apparent.]

6.   The limit is −6.25.

7.   Answers may vary as students begin to get the idea that the change in the definition of the function may be creating some problems, although not with the limit. This is a good checkpoint for the understanding of what it means to be a "limit."

8.
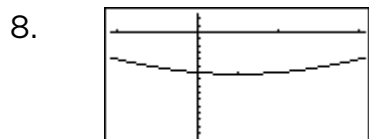
     [Note: the point becomes more visible.]

9.   All three limits are −6.25, although some students may try to refine the answers to longer decimals. This provides another good opportunity to stress the idea of "limit" as the value the function approaches.

10.   g(0.5) = −6, a value different from the limit.

11. Answers will vary; a good answer will include the fact that the function has a gap or a hole or a jump at the point of discontinuity. The idea is to have the students begin to think about the fact that simply having a limit does not guarantee the continuity of a function.